

# Dynamic Behaviour of Cracked Granite and Marble Columns Retrofitted with Steel Collars

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**SUMMARY** – *Marble and heavy stone columns are widely diffused in ancient churches and historical buildings in all the Mediterranean area. Their good mechanical properties allowed carrying great load values, while their bright colours and aesthetical characteristics have been used by a lot of ancient architects to achieve structural solutions with great visual impact.*

*Despite their good compressive strength, marble columns could be damaged from environmental effects (e.g. long-term effects or thermal loads), which could crack the structural members. In this way, the slenderness of the column increases drastically and the presence of an imposed ground shaking could be critical, since the column will be more vulnerable to rocking motion and to overturning risk.*

*This paper focuses on the rocking behaviour of cracked granite and marble columns subjected to a pulse type ground shaking. The effect of the presence of circular collars is analysed by means of a mechanical model. The overturning spectra are determined including the presence of the collars, showing their efficiency in reducing the overturning risk. Comparisons are shown with numerical analyses and a simplified design method is proposed.*

**Keywords:** *Marble columns, rocking motion, steel collars.*

## 1. Introduction

Marble and heavy stone columns are diffused structural members in the architectural heritage of the European area and their use is common since the ancient Roman age. Slender columns, made up with the local available stone, characterize several historical buildings, churches and monuments.

The features characterizing ancient stone columns can be drawn on the basis of different aspects of the structural member, such as material and geometry. Even if a great deal of these aspects exists, one feature that could be considered as common in almost all the marble and granite columns is their geometrical slenderness. The concepts of brightness, stateliness and greatness together with the good mechanical properties of granites and marbles have been used from architects in all historical ages taking advantage of tall and slender columns.

In fact, granites and heavy stones are characterized by good strength properties, which can be considered as function of their porosity. Ludovico Marques et al. (2012) carried out different uniaxial compressive tests

on granite and sandstone cylindrical specimens and demonstrated with a statistical correlation as the compressive strength and elastic modulus increased when the porosity decreased. Such consideration is based on the more marked heterogeneity and on the more frequent presence of inclusions, vacuums and micro-cracking in materials with high porosity. For values of porosity between 0.42% and 1.11% the experimentally determined strength was over 110 MPa, while the elastic modulus was always over 30000 MPa.

On the basis of such values, a monolithic stone column could be considered as a slender rigid member in most cases simply supported at its base.

The case of a column of width  $2b$  and height  $2h$ , subjected to a ground motion, with horizontal  $\ddot{u}_g$  and vertical  $\ddot{v}_g$  acceleration components was considered (Fig. 1a).

It is well-known from existing literature that a slender block subjected to a ground motion rocks if the overturning moment generated by the horizontal inertia force exceeds the restoring moment due to the weight of the block and vertical inertia force:

$$m\ddot{u}_g h = m(g + \ddot{v}_g)b \quad (1)$$

where  $m$  is the mass of the system,  $b$  and  $h$  are respectively the half width and half height of the block (Fig. 1a),  $\ddot{u}_g$  and  $\ddot{v}_g$  are the horizontal and the vertical components of ground acceleration.

The minimum acceleration inducing the rocking motion of the block can be determined by means of Eq. (1):

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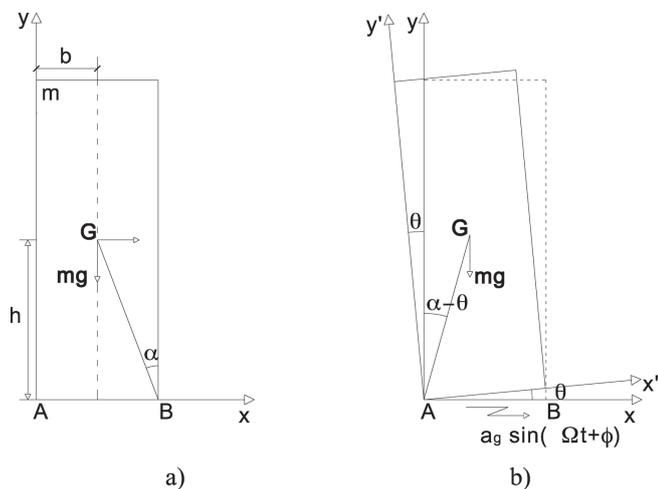


Fig. 1. A free-standing block subjected to a sine pulse. a) Free standing block; b) Rotated block under ground motion. Schema di un blocco rigido soggetto ad un'accelerazione alla base. a) Blocco rigido in configurazione stabile; b) Blocco ruotato soggetto ad un moto impresso al suolo.

$$\ddot{u}_g > g \left( 1 + \frac{\ddot{v}_g}{g} \right) \cdot \frac{b}{h} \quad (2)$$

If the vertical ground acceleration is neglected for sake of simplicity, and also considering that  $\tan \alpha = \frac{b}{h}$  and that for slender blocks  $\tan \alpha = \alpha$ , the minimum rocking acceleration results:

$$\ddot{u}_{g,\min} = g \cdot \frac{b}{h} = \alpha \cdot g \quad (3)$$

Eq. (3) means that the activation of the rocking motion on a column strictly depends on its geometric characteristics, and the value of the minimum rocking ground acceleration decreases as the slenderness increases.

Considering the most common values of slenderness of existing columns (e.g. between 12 and 40 as stated in Campione et al. (2014)) and also considering that a large percentage of the historical architectural heritage is built in seismic areas, it emerges as these vertical structural members are particularly vulnerable to the rocking motion. The activation of the dynamic motion could lead either to overturning of the column or to local material crushing due to the stress concentration in the bottom zones near the edges. An example of a damaged column due to its rocking motion under a seismic motion is shown in Fig. 2.

It refers to a column of the City Hall building in Mirandola (Italy), which was subjected to the severe earthquake (6.1 Mw) occurred in northern Italy during the month of May 2012. As could be noted, the slender column rocked and was damaged in the corner near one edge due to the stress concentration, which exceeds the compressive strength of the pink marble.

The effect of a ground shaking could be more critical and dangerous in columns which are already cracked due to different environmental or external effects. Such effects could be related to thermal actions,



Fig. 2. Heavy stone column in the City Hall of Mirandola (Italy) damaged after the Emilia earthquake during May 2012. Colonna in marmo nel Municipio di Mirandola (Italia) danneggiata dopo l'evento sismico dell'Emilia (Maggio 2012).

or initial imperfections. The weak nature of stones and their low tensile strength lead to the quick propagation of micro-cracks, which could seriously damage the structural member. As a consequence the geometric slenderness increases and considering Eq. (3) the minimum overturning acceleration drastically decreases.

One of the most common techniques adopted to retrofit damaged cracked columns, especially during the post-earthquake phase and in case of wide cracks, is the application of steel collars (Campione et al. (2014)). Even if such a technique is very commonly used in the engineering practice, its effects are not completely known and poor guidelines are provided in international Codes. Number, dimensions and mechanical properties of the strengthening devices are often decided based on professional practice and after suggestions from restoration manuals, without the total perception of the inner physical phenomena. For example, the theoretical justification for the application of steel collars in granite and heavy stone columns is often associated to the confinement action applied by the collars, as experienced in reinforced concrete columns. It has to be noted that granites and heavy stones are materials having low Poisson's coefficient (0.1-0.2), and consequently passive confinement is of questionable efficacy, particularly when the cracked configuration makes the lateral expansion completely altered. Therefore, applications of the retrofit technique, designed to improve strength and ductility of marble columns by passive confinement, can likely result to be inefficient.

In this paper the effect of steel collars on the dynamic behaviour of cracked granite and heavy stone columns subjected to a sine pulse is analyzed and treated with the classical tools of the theory of rocking of rigid blocks [Housner GW, 1963]. The equation of motion governing the rocking behaviour is adapted to the case study, and the effect of the steel collars is determined in terms of overturning spectra. Obtained equations are implemented in a computer code and results are compared with numerical analyses carried out

with a commercial software [Working Model 2D 2000]. It has to be noted that a lot of algorithms and theories are actually available to study the rocking behaviour of rigid blocks. However in this work the Housner's theory was used and adapted to the case study for sake of simplicity and with the goal to provide useful and simple design guidelines.

## 2. Theoretical background

The theory of [Housner GW, 1963] represents the first significant study on the dynamic response of a rigid block supported on its base and undergoing horizontal accelerations. The author examined the case of free and forced oscillations responses to rectangular and sine pulse, describing the motion of rocking blocks through an equation of motion.

The case study refers to a single block with an initial rotation angle  $\theta(0) = \theta_0$  and angular velocity  $\dot{\theta}(0) = \dot{\theta}_0$  (both to be considered only theoretically). Under a positive horizontal ground acceleration (Fig. 1b) and assuming that the static friction coefficient between block and ground is large enough to prevent sliding, the block rotates with a negative rotation  $\theta$  when the ground acceleration amplitude reach the value of  $\ddot{u}_{g,min}$ . The equations that govern this motion are:

$$\begin{aligned} J_0 \dot{\theta}(t) + mgR \cdot \sin[-\alpha - \theta(t)] &= \\ &= -m\ddot{u}_g(t)R \cos[-\alpha - \theta(t)], \quad (4) \\ \theta(t) &< 0 \end{aligned}$$

$$\begin{aligned} J_0 \dot{\theta}(t) + mgR \cdot \sin[\alpha - \theta(t)] &= \quad (5) \\ &= -m\ddot{u}_g(t)R \cos[\alpha - \theta(t)], \\ \theta(t) &> 0 \end{aligned}$$

where  $R$  is the distance between point A and the center of gravity of the block (see Fig. 1),  $J_0$  is the moment of inertia of the block around point A and is equal to  $4/3mR^2$  for rectangular blocks.

The Eqs. (4) and (5) are well-known in literature and were analyzed by several authors [Yim et al., 1980; Zhang and Makris, 2001; Makris and Zhang, 2001; Jacobsen and Ayre, 1958; Kounadis, 2010] in order to determine the response of rigid blocks under different ground excitations. It has to be noted that among the different kinds of ground motions, the sine pulse type is one of the most studied in the literature [Housner GW, 1963; Yim et al., 1980; Zhang and Makris, 2001; Makris and Zhang, 2001; Jacobsen and Ayre, 1958; Kounadis 2010; Konstantinidis and Makris, 2010; Ferrero and Perry, 2004]. Such type of ground motion was studied in the present work because, as shown by [Makris and Zhang 2001], it is able to approximate the kinematic characteristics of several recorded ground motions during real earthquakes.

Fig. 3 shows the comparisons between three different time histories relative to the events of Friuli (1976), Aigion (1995) and Chi-Chi (1999) and the simplified sine pulse representation, determined as expressed in [Jacobsen and Ayre, 1958]. As could be noted and as

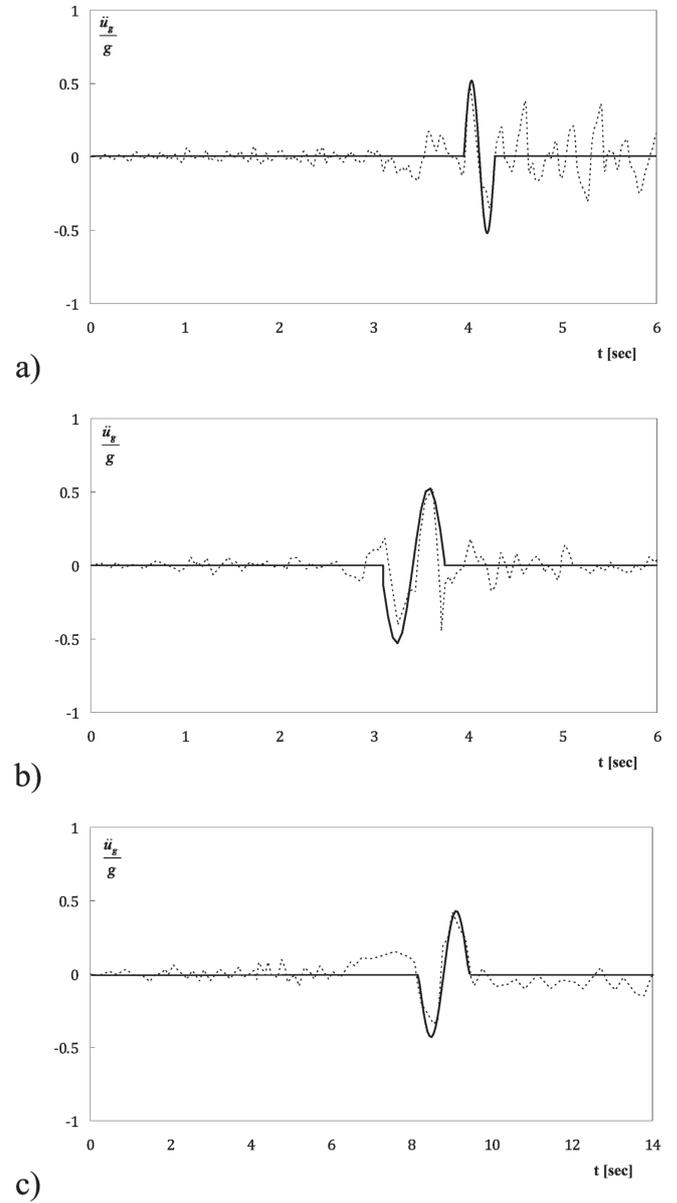


Fig. 3. Comparison between acceleration time histories and the representative sine pulses. a) Friuli (1976); b) Aigion (1995); c) Chi-Chi (1999).

Confronto tra le storie temporali di alcune accelerazioni al suolo relative ad eventi sismici reali e le pulsazioni sinusoidali equivalenti. a) Friuli (1976); b) Aigion (1995); c) Chi-Chi (1999).

already shown in [Makris and Zhang, 2001] the simple one-sine pulse is able to approximate some of the recorded time histories of real earthquakes.

Due to the selection of a sine pulse as horizontal input the function  $\ddot{u}_g(t)$  assumes the following form:

$$\ddot{u}_g(t) = a_{p0} \sin(\Omega t + \phi) \text{ for } -\frac{\phi}{\Omega} \leq t \leq T_{ex} \quad (6a)$$

$$\ddot{u}_g(t) = 0 \text{ otherwise} \quad (6b)$$

where  $T_{ex}$  is the period of the external sine pulse equal to  $\frac{(2\pi - \phi)}{\Omega}$ ,  $a_{p0}$  is the peak ground acceleration,  $\Omega$

is the frequency of the sine pulse and  $\phi$  is the phase amplitude.

If the hypothesis of slender block ( $\frac{b}{h} < 0.1$ ) is made together with the assumption of small oscillations, Eq. (4) and (5) could be linearized (Zhang and Makris, 2001), considering that the sine of the angle can be approximated by the angle. In this way, the governing equations of motion become:

$$\begin{aligned} \ddot{\theta}(t) - p^2\theta(t) &= -\frac{a_g}{g}p^2\sin(\Omega t + \phi) + p^2\alpha & (7a) \\ \theta(t) &< 0 \end{aligned}$$

$$\begin{aligned} \ddot{\theta}(t) - p^2\theta(t) &= -\frac{a_g}{g}p^2\sin(\Omega t + \phi) - p^2\alpha & (7b) \\ \theta(t) &> 0 \end{aligned}$$

where  $p$  is the frequency parameter representing a measure of the dynamic characteristics of the rocking block, which can be calculated as  $\sqrt{\frac{mgR}{J_0}}$ . As stated by Housner, despite the fact that oscillation frequency of a rigid block is not constant since it actually depends on the vibration amplitude, nevertheless the parameter  $p$  can be considered as a rough measure of the dynamic characteristics of the block. It tends to small values for more slender blocks.

The integration of the ordinary differential equations expressed by Eqs. (7) gives:

$$\begin{aligned} \theta(t) &= A_1 \sinh(pt) + A_2 \cosh(pt) - \\ &- \alpha + \frac{1}{1 + \frac{\Omega^2}{p^2}} \frac{a_g}{g} \sin(\Omega t + \phi) & (8a) \\ \theta(t) &< 0 \end{aligned}$$

$$\begin{aligned} \theta(t) &= A_3 \sinh(pt) + A_4 \cosh(pt) + \\ &+ \alpha + \frac{1}{1 + \frac{\Omega^2}{p^2}} \frac{a_g}{g} \sin(\Omega t + \phi) & (8b) \\ \theta(t) &> 0 \end{aligned}$$

where the four constants can be written as:

$$A_1 = A_3 = \frac{\dot{\theta}_0}{p} - \frac{\Omega/p}{1 + \frac{\Omega^2}{p^2}} \frac{a_g}{g} \cos(\phi) \quad (9a)$$

$$A_2 = \theta_0 + \alpha - \frac{1}{1 + \frac{\Omega^2}{p^2}} \frac{a_g}{g} \sin(\phi) \quad (9b)$$

$$A_4 = \theta_0 - \alpha - \frac{1}{1 + \frac{\Omega^2}{p^2}} \frac{a_g}{g} \sin(\phi) \quad (9c)$$

Eqs. (8) allow to construct the time history of the rocking response of a block under a sine pulse, and are valid when  $t < T_{ex}$ .

When  $t > T_{ex}$  the block enters in free oscillation regime, with initial conditions calculated by Eqs. (8) for  $t = T_{ex}$ .

Within the limits of the linear approximation, the equations of rocking motion are particularized in the following form:

$$\begin{aligned} \ddot{\theta}(t) - p^2\theta(t) &= -p^2\alpha, & (10a) \\ \theta(t) &< 0 \end{aligned}$$

$$\begin{aligned} \ddot{\theta}(t) - p^2\theta(t) &= p^2\alpha, & (10b) \\ \theta(t) &> 0 \end{aligned}$$

Eqs. (10) have to be integrated considering the initial conditions  $\theta(0) = \theta_0(T_{ex})$  and  $\dot{\theta}(0) = \dot{\theta}(T_{ex})$ , meaning that the initial conditions when the block enters in free oscillation regime are those relative to the end of the sine pulse:

$$\theta(t) = \alpha - (\alpha - \theta(T_{ex})) \cosh(pt) + \frac{\dot{\theta}(T_{ex})}{p} \sinh(pt) \quad (11a)$$

$$\theta(t) < 0$$

$$\theta(t) = \alpha + (\alpha + \theta(T_{ex})) \cosh(pt) + \frac{\dot{\theta}(T_{ex})}{p} \sinh(pt) \quad (11b)$$

$$\theta(t) > 0$$

Eqs. (8) and (11) can be assembled in order to draw the response time history of the block subjected to a sine pulse, considering also the reduction of the angular velocity when the block impacts the ground. The latter can be taken into account simply considering that at every impact the ratio between the velocity after and before is equal to the restitution coefficient  $\nu$  proposed by [Housner GW, 1963] in the following form:

$$\nu = 1 - 2 \frac{mR^2\alpha}{J_0} \quad (12)$$

It has to be noted that when  $\dot{\theta}(T_{ex})$  is assumed equal to 0, the rocking period  $T$  can be calculated by setting the right end side of Eq. (11) equal to 0. The rocking period  $T$  represents the time needed by the block under free oscillations to move from the starting position  $\theta(0) = \theta_0$  to the vertical position  $\theta(T) = 0$ , and under the hypothesis of  $\nu = 1$ . This value is expressed by:

$$T = \frac{1}{p} \cosh^{-1} \left( \frac{1}{1 - \theta_0/\alpha} \right) \quad (13)$$

### 3. Case study: two equal blocks connected with springs

The above mentioned motion equations (Eq. (7) and Eq. (10)) have to be updated when considering the case of a block subdivided in two equal halves connected with steel collars.

When the rigid column is subjected to a vertical crack (Figs. 4a and 4b) the slenderness of the member drastically increases, which means that the frequency parameter  $p$  assumes small values. Consequently, the rocking period  $T$  expressed by Eq. (13) increases, meaning that the block sustains the motion for a longer time.

It has to be clarified that the assumption of element cracked all along its height and for its full depth is a conservative hypothesis. It is, in fact, the most critical hypothesis for the study of overturning conditions.

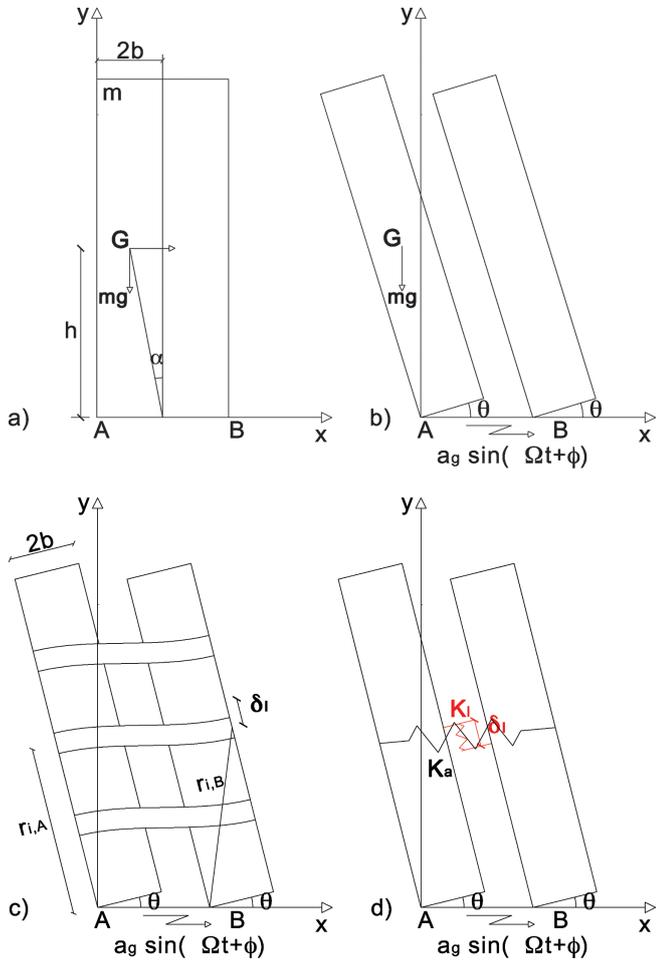


Fig. 4. Cracked column subjected to a sine pulse. a) Free standing subdivided column; b) Column without interposed spring; c) Effect of the collars; d) Model of column with interposed spring.  
 Colonna fessurata soggetta ad una pulsazione sinusoidale. a) Colonna fessurata in configurazione stabile; b) Colonna senza rinforzi; c) Effetto delle cerchiature metalliche; d) Modello della colonna con rinforzi.

For conditions of only partially cracked members the analysis of the interacting forces between the bodies is needed and will be addressed in future studies.

The collars are mainly subjected to normal force and shear if an equal rotation is imposed to the blocks (see Fig. 4c). Consequently their effectiveness can be related to their capacity in reducing the relative sliding between the two blocks. On these basis, the presence of a collar on the behaviour of a rocking column could be modelled in a schematic way with two springs, representing respectively the axial  $K_a$  and the lateral  $K_l$  stiffness of the collar (Fig. 4d). These can be calculated as:

$$K_a = \frac{E \cdot A_c}{l_c} \quad (14)$$

$$K_l = \frac{12 \cdot E \cdot I_c}{l_c^3} \quad (15)$$

being  $E$  the elastic modulus of steel,  $A_c$  the area of the transverse section of the collar,  $I_c$  its moment of inertia and  $l_c$  the circumferential length of the collar.

The analysis has to be carried out with a step-by-step algorithm, calculating the distortion in the springs, and consequently their reactions, under the hypothesis of linear elastic behaviour.

At the first step of the analysis, the solution provided by Eqs. (11) allows computing the value of  $\theta(t_1) = \theta_1$ . At the next integration step the reaction force in the horizontal spring could be calculated by determining its axial elongation:

$$\delta_{a,1} = 4b - d_1 \quad (16)$$

where  $4b$  is the original un-deformed length of the spring and  $d_1$  the length corresponding to the deformed shape of the collar at the first step of the analysis.

The latter can be computed with the following equation:

$$d_1 = \sqrt{\left[ \left( -2r_{i,A} \sin\left(\frac{\theta_1}{2}\right) \cdot \sin(\beta) \right) - \left( 4b - 2r_{i,B} \sin\left(\frac{\theta_1}{2}\right) \sin(180 - \beta - \gamma) \right) \right]^2 + \left[ -2r_{i,A} \sin\left(\frac{\theta_1}{2}\right) \cos(\beta) - 2r_{i,B} \sin\left(\frac{\theta_1}{2}\right) \cos(180 - \beta - \gamma) \right]^2} \quad (17)$$

where  $r_{i,A}$  and  $r_{i,B}$  are the distance of the left and right edge of the  $i$ -th collar from points A and B, respectively (see Fig. 4c). Angles  $\beta$  and  $\gamma$  can be calculated as:

$$\beta = \arcsin\left(\cos\frac{\theta_1}{2}\right) \quad (18)$$

$$\gamma = \arctan\left(\frac{2b}{h}\right)$$

The lateral displacement for each collar can be more easily calculated as:

$$\delta_{l,1} = 4b \cdot \tan\frac{\theta_1}{2} \quad (19)$$

At the next integration step the equation of motion is written including the effect of the reaction force exerted by the springs:

$$J_0 \ddot{\theta}(t) + mgR \cdot \sin[-\alpha - \theta(t)] + F_{l,1} \cdot 2b - F_{a,1} \cdot h = -m\ddot{u}_g(t)R \cos[-\alpha - \theta(t)], \quad (20a)$$

$$\theta(t) < 0$$

$$J_0 \ddot{\theta}(t) + mgR \cdot \sin[\alpha - \theta(t)] - F_{l,1} \cdot 2b + F_{a,1} \cdot h = -m\ddot{u}_g(t)R \cos[\alpha - \theta(t)], \quad (20b)$$

$$\theta(t) > 0$$

where  $F_{l,1}$  and  $F_{a,1}$  are the reaction forces at the first step of the vertical and horizontal spring, respectively, calculated as:

$$F_{a,1} = K_a \delta_{a,1}$$

$$F_{l,1} = K_l \delta_{l,1} \quad (21).$$

The solution of Eqs (20) allows calculating the value of  $\theta(t_2) = \theta_2$ . If this procedure is repeated, the time history of an half block with interposed springs can be computed.

#### 4. Implementation and comparison with numerical analysis

The above mentioned differential equations are implemented in a computer code in the Mathematica environment [Wolfram, 1999] and numerically solved.

For the validation of the solution comparisons are made with numerical results obtained by using [Working Model 2D 2000]. The latter is a software which includes robust numerical techniques and sophisticated editing capabilities. This software was selected because allowed a complete modelling of the dynamic motion of rigid bodies. Furthermore it is widely validated in literature and used to analyse experimental results [Konstantinidis and Makris, 2010; Fierro and Perry, 2004; Stephen Wolfram, 1999]. It allows computing the response of mechanically interacting rigid bodies under several constraints and forces, displacements or accelerations.

One of the most challenging tasks in the dynamic simulation of rigid bodies is the treatment of the contact interfaces. In [Working Model 2D, 2000], the satisfaction of all imposed constraints at the contact interfaces is enforced simultaneously during the numerical integration. In the tangential direction, the contact interface of adjacent bodies is modelled by static and kinetic Coulomb friction. Regardless of whether there is sliding motion or not, the rigid body that models the column, while engaging in rocking motion, can impact the rigid body that models the ground. During an integration step, two colliding bodies may overlap by a small amount.

The numerical integration of the equations of rocking motion together with the satisfaction of the constraint conditions (friction and restitution), is performed using a robust Kutta-Merson method (5th order Runge-Kutta). The integration error as well as the model assembly and the collision overlap tolerances can be set to achieve the desired precision. With the available variable-time step Kutta-Merson scheme, near collision the time step is reduced appropriately to restrict the overlap between bodies from exceeding the specified overlap tolerance. For all the numerical analyses carried out in the present work, the overlap error tolerance was set to  $10^{-5}$  cm as suggested in Konstantinidis and Makris (2010).

Analyses are first carried out for the case of a free standing block studied by Zhang [Zhang and Makris, 2001], in order to validate the implementation in Mathematica [Wolfram, 1999]. The block has parameter  $p = 2.14$  rad/sec,  $\alpha = 0.25$  rad and  $\nu = 0.9$  and was subjected to a sine pulse having  $\Omega/p = 5$  and different amplitudes. Fig. 5 shows the comparison between the

time histories in terms of normalized angle  $\theta/\alpha$  (ratio between rotation angle and slenderness) and normalized angular velocities (ratio between angular velocity and frequency parameter) obtained with Mathematica and Working Model. It could be noted that the two approaches lead to the same solution, matching also the theoretical response obtained by Zhang (not reported for sake of clarity). It can also be observed that a block can survive a pulse with acceleration amplitude larger than the minimum acceleration pulse necessary to overturn it. This finding was also reported by Zhang.

The same case study was later analysed with sine pulses of increasing amplitudes (Figs. 5b, 5c and 5d). The response of a solid block is compared in Fig. 6 with the response of a half block and of a half block with a vertical spring of stiffness  $K_1 = 50$  N/m. The solid block has width double the one reported in Fig. 5. The figure shows the effectiveness of the spring in restraining the block, avoiding overturning even if large amplitudes of rotation are reached. The behaviour of the half block with or without the spring is similar at the beginning of the record but when large rotations are introduced the force exerted by the spring is able to restrain the block from overturning, as already shown in [Zhang and Makris, 2001].

The effect of the steel collars in reducing the overturning risk is more evident by considering the spectrum shown in Fig. (7). The minimum overturning pulse amplitude  $\alpha_{p0}$ , normalised with respect to  $ag$  is plotted as a function of the  $\Omega/p$  ratio. Such a representation is well-known from literature [Zhang and Makris, 2001; Makris and Zhang, 2001; Kounadis 2010; Konstantinidis and Makris, 2010; Fierro and Perry, 2004; Stephen Wolfram, 1999], and was theoretically explained in [Zhang and Makris, 2001]. Fig. (7a) represents the overturning spectrum shown by Zhang which demonstrated as overturning can occur after or before the block experienced an impact with the ground. As could be observed from Fig. (7b) the presence of the vertical spring with  $K_1 = 50$  N/m is able to reduce the overturning area, and higher values of acceleration amplitude are needed to overturn the column. In particular, the presence of the vertical spring is more efficient when the frequency increases. In this range the minimum overturning acceleration (without impact) increases up to three times the value obtained for the configuration without spring. Moreover, the maximum frequency threshold for non-overturning, in case of impact, decreases of about 26%, demonstrating that the block can also sustain lower frequency pulses.

#### 5. Simplified design procedure

Theoretical and numerical analyses have shown as collars are able to limit the overturning risk. For design purposes, it has to be noted that if the scheme of Fig. (4c) is assumed, the maximum displacement which can occur in the vertical spring is equal to:

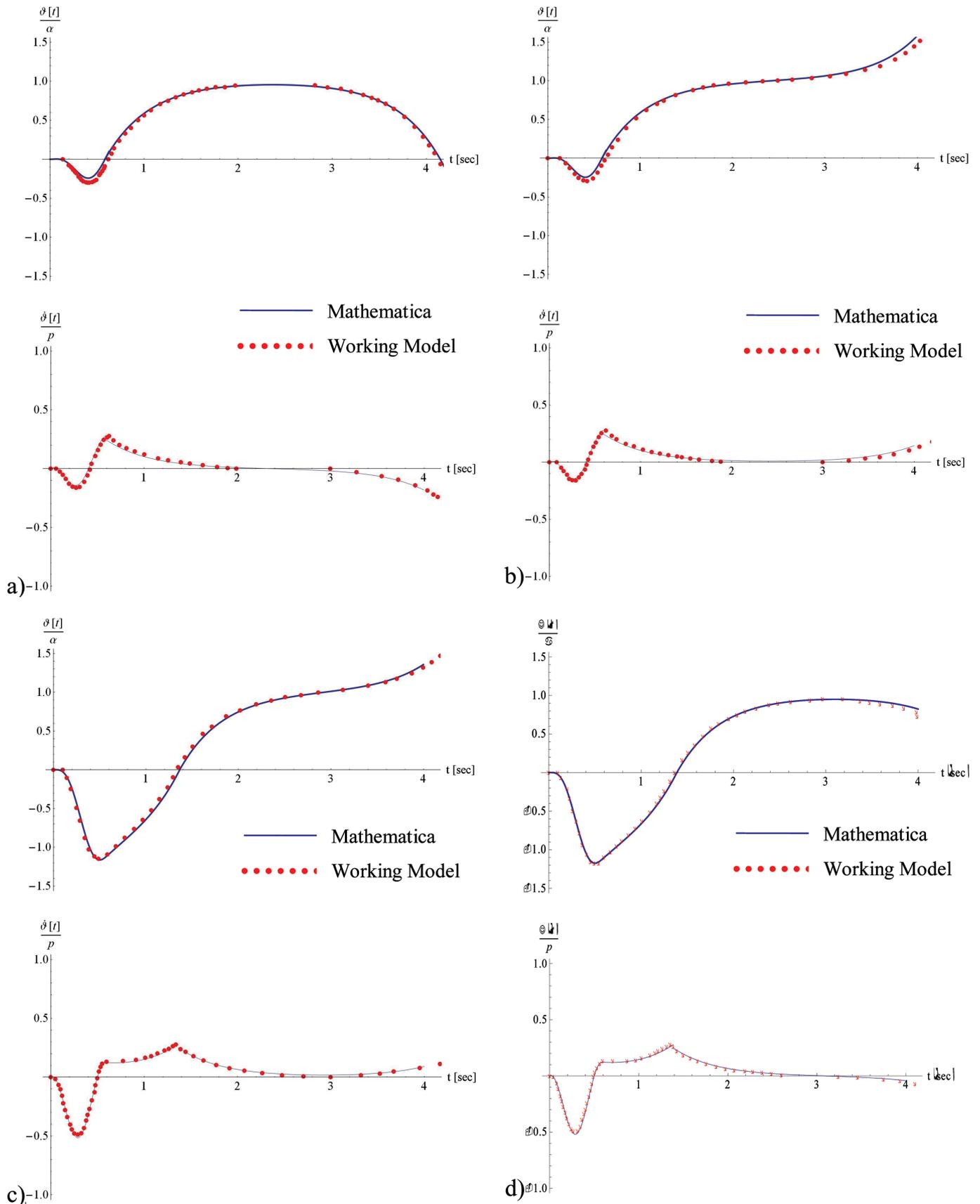


Fig. 5. Comparison between the analytical response obtained with Mathematica and the numerical analysis (Working Model) for a free standing block subjected to a sine pulse. a)  $a_g = 3\alpha_g$ ; b)  $a_g = 3.01\alpha_g$ ; c)  $a_g = 6.32\alpha_g$ ; d)  $a_g = 6.33\alpha_g$ .

Confronto tra la risposta analitica ottenuta con Mathematica e i risultati numerici (Working Model) per un blocco appoggiato soggetto ad una pulsazione sinusoidale a)  $a_g = 3\alpha_g$ ; b)  $a_g = 3.01\alpha_g$ ; c)  $a_g = 6.32\alpha_g$ ; d)  $a_g = 6.33\alpha_g$ .

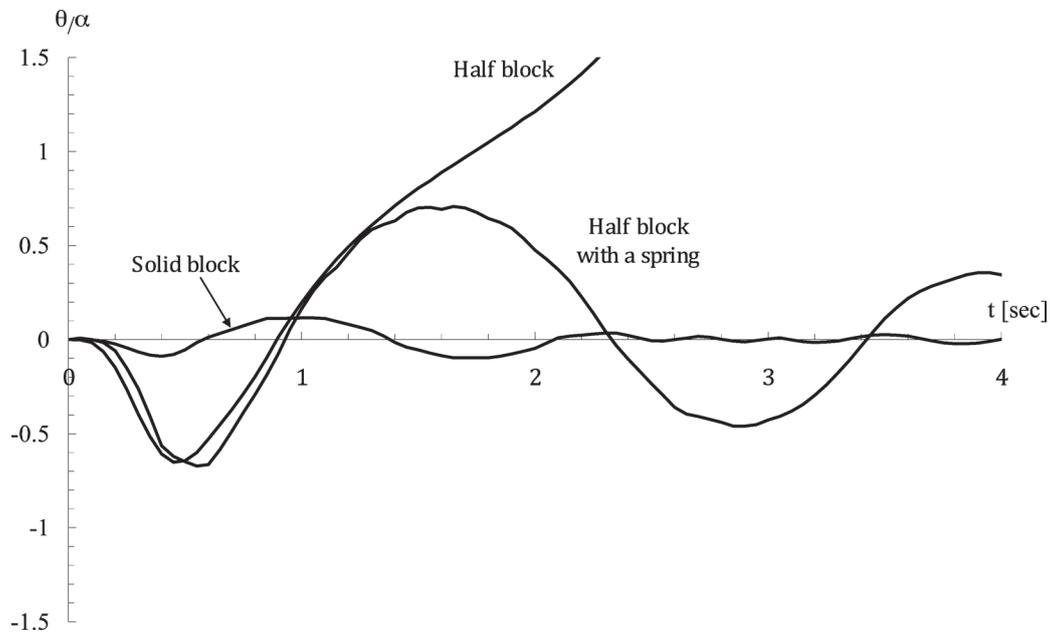


Fig. 6. Comparison of the normalised time histories of a solid block, half block and the latter with a spring subjected to a sine pulse having  $\Omega/p = 5$  and amplitudes equal to  $6.32\alpha g$ .

Confronto tra le storie temporali normalizzate della rotazione di un blocco intero, blocco fessurato, blocco fessurato e rinforzato, soggetti ad una pulsazione sinusoidale di frequenza normalizzata  $\Omega/p = 5$  e ampiezza pari a  $6.32\alpha g$ .

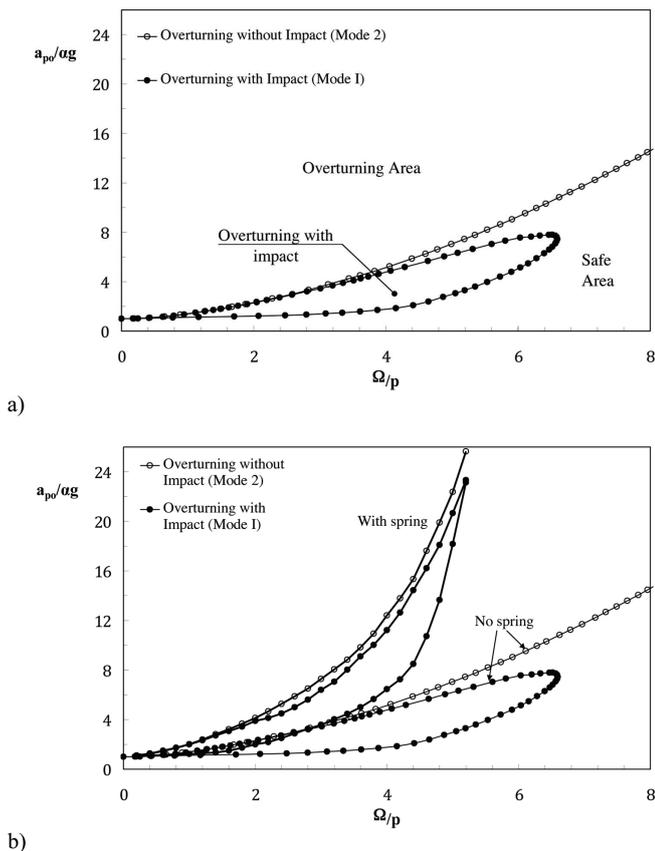


Fig. 7. Overturning spectra for the block analysed in [Zhang and Makris (2001)]; a) Free standing block; b) Free standing block with a vertical spring ( $K_1 = 50 \text{ N/m}$ ).

Spettri di ribaltamento per il blocco analizzato in [Zhang and Makris (2001)]; a) Blocco semplice; b) Blocco rinforzato con molla verticale ( $K_1 = 50 \text{ N/m}$ ).

$$\delta_{l,y} = \frac{M_y (4b)^3}{12EI_c} \quad (22)$$

being  $M_y$  the yield moment  $= \frac{f_y \cdot t_c \cdot h_c^2}{6}$ .

The rotation angle corresponding to this spring elongation can be calculated as:

$$\theta_y = 2 \arctan\left(\frac{\delta_{l,y}}{4b}\right) \quad (23)$$

From another point of view, it has to be noted that the overturning risk increases for low frequency pulses. As noted in [Makris and Zhang, 2001], when the ratio  $\Omega/p$  is near to zero, the column is subjected to a very long duration pulse, and when the latter is near its peak the ground acceleration can be considered quasi-constant. Under this assumption, if the spring exerts its maximum force, the balance of moments can be written as:

$$\begin{aligned} m \cdot a_g \cdot R \cos(\alpha - \theta_y) &= \\ &= mg \sin(\alpha - \theta_y) + k \cdot \delta_y 2b \end{aligned} \quad (24)$$

where  $m$  is the mass of the block,  $a_g$  is the design peak ground acceleration,  $\alpha$ ,  $R$  and  $2b$  are the geometrical characteristics of the column (Fig. 5a),  $\theta_y$  is defined by Eq. (23) and  $k$  is the minimum required stiffness. Eq. (24) allows calculating in a simplified manner the minimum required stiffness, compatible with the strength features of the steel collars.

Fig. (8) shows the minimum required stiffness, calculated by Eq. (24), as a function of the pulse amplitude normalized to  $g$  and the angle  $\alpha$ , for steel col-

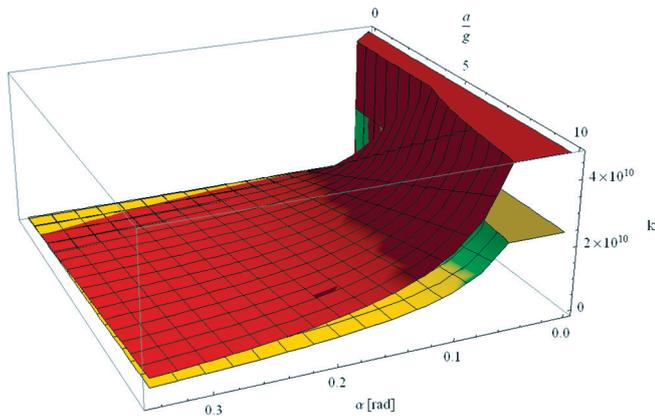


Fig. 8. Minimum spring stiffness as a function of the  $\alpha$  angle and the normalised pulse amplitude  $a_g/g$ .

Rigidizza minima richiesta alla molla al variare dell'angolo  $\alpha$  e dell'accelerazione di picco normalizzata  $a_g/g$ .

lars with  $E = 206000 \text{ MPa}$  and  $f_y = 235 \text{ MPa}$ . The two surfaces are for a member with  $b = 1.0 \text{ m}$  and specific weight of  $2000 \text{ kg/m}^3$  (red surface) and  $1000 \text{ kg/m}^3$  (yellow surface). It can be noted that for low pulse amplitude and shorter columns, the required stiffness is quite low and the two functions are almost matching. For more slender blocks and greater acceleration amplitude the minimum stiffness assumes higher values, particularly for the heavier column.

## 6. Conclusions

The dynamic behaviour of rigid columns subjected to an imposed pulse type ground motion is analysed. In particular, columns are here considered as completely separated by a vertical crack. The effect of the application of steel collars was studied. Collars were modelled with elastic springs, placed in horizontal and vertical direction, simulating respectively the axial and the lateral stiffness of the collars. A step-by-step analysis was carried out by solving numerically the equations of motion and results were validated with numerical analyses performed with the software Working Model. From the obtained results the following conclusions can be drawn:

- granite and heavy stone columns are vulnerable to the overturning risk due to their slenderness. Such a risk increases when the column is cracked along its height;
- steel collars restrain the relative sliding between two adjacent blocks, limiting the amplitude of oscillations;
- a minimum number of collars is able to reduce the overturning risk; number and area of collars depend on the slenderness of the block and from the weight of the member.

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# Comportamento dinamico di colonne in marmo rinforzate con cerchiature metalliche

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Le colonne in marmo e in granito rappresentano un elemento strutturale molto diffuso nell'area del bacino Mediterraneo. Sin dall'epoca romana una grande quantità di edifici storici, chiese e monumenti furono costruiti sfruttando sia le proprietà estetiche che meccaniche di colonne snelle costruite con il materiale lapideo locale.

In particolare, il granito ed il marmo sono materiali dotati di buona resistenza a compressione, la quale può essere messa in relazione alla porosità della pietra. I risultati sperimentali portati avanti da [Ludovico Marques et al, 2012] dimostrarono che la resistenza a compressione e il modulo elastico incrementa quando la porosità diminuisce. Questa considerazione è basata sulla eterogeneità, sulla presenza di inclusioni, vuoti e micro fessure nei materiali con grande porosità.

Da queste considerazioni può dedursi come una colonna in marmo può essere considerata come un blocco rigido, dotato di una propria snellezza geometrica e pertanto vulnerabile a delle rotazioni di corpo rigido (moto di rocking) derivanti da accelerazioni impresse al suolo. L'attivazione del moto di rocking su una colonna dipende rigorosamente dalle sue caratteristiche geometriche e il valore minimo dell'accelerazione del terreno che produce il moto diminuisce se la snellezza incrementa. In condizioni di crisi può verificarsi una eccessiva rotazione della colonna o lo schiacciamento locale del materiale dovuto ad una concentrazione di sforzi nelle zone di base vicino ai bordi. Inoltre si può osservare che l'effetto del moto del terreno può essere maggiormente critico e dannoso nelle colonne che, per cause ambientali o esterne, presentano già delle fessure. La bassa resistenza a trazione delle pietre porta ad una veloce propagazione delle micro fratture che può seriamente danneggiare l'elemento strutturale. Come conseguenza, l'incremento della snellezza geometrica dovuto alla fessura assiale produce una drastica diminuzione dell'accelerazione critica che produce il ribaltamento.

Quando si presentano colonne fessurate assialmente, una delle più comuni tecniche adottate per migliorare il comportamento strutturale, specialmente nella fase post sisma e nei casi di presenza di ampie fessure, è l'applicazione di cerchiature metalliche. Quest'ultime vengono spesso applicate all'elemento strutturale con l'obiettivo di migliorarne la resistenza o la duttilità mediante l'effetto di confinamento; tuttavia, il basso modulo di Poisson e la fessurazione che modifica l'espansione laterale rende poco efficace l'effetto confinante. Risulta quindi di interesse pratico riconoscere se l'applicazione di cerchiature metalliche può avere effetti su altri aspetti strutturali, come per esempio sul comportamento dinamico della colonna.

In questo articolo si presenta uno studio condotto sull'effetto delle cerchiature metalliche sul comportamento dinamico di colonne in granito o pietra fessurate e soggette ad un'accelerazione alla base di tipo impulso

sinusoidale. Il moto della colonna viene descritto dalle equazioni di governo del moto di rocking proposte da [Housner, 1963], sotto l'ipotesi di modello bidimensionale e assenza di scorrimenti blocco-terreno (coefficiente di attrito statico terreno-blocco sufficientemente elevato).

Le equazioni del moto vengono quindi adattate al caso studio, ovvero schematizzando la colonna fessurata come due blocchi uguali e adiacenti. Se si ipotizza che i punti di estremità delle cerchiature rimangano fisse, da semplici considerazioni geometriche si osserva che i rinforzi sono soggetti ad una compressione e ad uno spostamento laterale e conseguentemente il loro effetto può essere messo in relazione alla loro capacità di ridurre lo scorrimento relativo tra i due blocchi. Sulla base di questa considerazione la presenza delle cerchiature sul comportamento al rocking della colonna può essere modellato in modo schematico con due molle che rappresentano rispettivamente la rigidezza assiale e quella a taglio della cerchiatura. Le analisi sono state condotte con un algoritmo step-by-step calcolando le distorsioni delle molle e conseguentemente le loro reazioni sotto il comportamento elastico lineare, ed inserendo quest'ultime nelle equazioni del moto.

Le equazioni differenziali trattate sono implementate in ambiente Mathematica [Wolfram, 1999]. Per la validazione della soluzione sono stati fatti dei confronti con i risultati numerici ottenuti usando il software commerciale [Working Model 2D 2000] che permette una modellazione completa nei riguardi del moto dinamico dei corpi rigidi. Inizialmente le analisi sono condotte per il caso di blocco libero allo scopo di validare gli algoritmi adottati con dati presenti in letteratura. Quindi si valuta il comportamento sia nel caso di presenza che in assenza di molla. Confrontando i risultati tra il caso di colonna libera e colonna rinforzata si nota come il rinforzo eserciti la sua azione solo quando il blocco subisce elevate rotazioni, e la sua reazione è capace di trattenere il blocco che altrimenti tende a ribaltare, come già mostrato da [Zhang and Makris, 2001]. Le analisi condotte al variare sia dell'ampiezza che della frequenza della forzante applicata, hanno permesso di costruire gli "spettri di ribaltamento", così come definiti in [Zhang and Makris, 2001]. Questi forniscono la variazione dell'accelerazione critica che produce il ribaltamento al variare della frequenza della forzante. Dall'andamento delle curve emerge come l'applicazione dei rinforzi risulti più efficace per valori elevati di frequenza.

Infine, viene proposta una procedura che permette di calcolare in maniera semplificata la rigidezza minima richiesta compatibile con le caratteristiche di resistenza delle cerchiature. Si nota come per impulsi con ampiezza più piccola e colonne più corte la rigidezza richiesta è abbastanza bassa. Per blocchi maggiormente snelli e accelerazioni più grandi la rigidezza minima assume valori più elevati specialmente per colonne più pesanti.